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SELECTED PROBLEMS

Classical Electrodynamics

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SLEZSKÁ UNIVERZITA V OPAVĚ

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ELECTROMAGNETIC FIELD - Problems

1 *Motion of a charged non-relativistic particle in electric and magnetic field.* Solve equations of motion for a non-relativistic charged particle with mass m and charge e moving a) in a constant homogeneous electric field $\boldsymbol{E} = (E, 0, 0)$ (initial conditions $\boldsymbol{r} = 0$, $\dot{\boldsymbol{r}} = \boldsymbol{v}_0$ for $t = 0$); b) in a constant homogeneous magnetic field $\boldsymbol{B} = (0, 0, \mathrm{B})$ (initial conditions $\boldsymbol{r} = 0$, $\dot{\boldsymbol{r}} = (\boldsymbol{v}_0, 0, 0)$).
[a) $x = \frac{eE}{2m}\ t^2\ + v_{0x}t,\ y = v_{0y}t,\ z = v_{0z}t$; b) $x = \frac{v_0}{\alpha}\ \sin\alpha t,\ y = -\frac{v_0}{\alpha}(1 - \cos\alpha t),\ z = 0;\ \alpha = \frac{eB}{m}$]

2 *Žáček's magnetron.* Demonstrate that the trajectory of a particle of mass m and charge e in crossed constant homogeneous fields $\boldsymbol{E} = (0, E, 0)$, $\boldsymbol{B} = (0, 0, B)$ is a cycloid, if initially $\boldsymbol{r}\,(0) = (0,0, z_0)$, $\dot{\boldsymbol{r}}\,(0) = 0$. This cycloid is generated by rolling a circle with radius $r_0 = mE/eB^2$ on surface $z = z_0$ along the x-axis with the angular frequency $\omega_c = eB\,/\,m$ (the cyclotron frequency).
[$x = \frac{E}{\omega_c B}(\omega_c t - \sin\omega_c t),\ y = \frac{E}{\omega_c B}(1 - \cos\omega_c t),\ z = z_0$]

3 *Normal Zeeman effect.* Electron of mass $m \doteq 9{,}1.10^{-31}$ kg and charge $e \doteq\ -1, 6.10^{-19}$ C bound to the origin by force $-k\boldsymbol{r}$ is harmoniously oscillating with angular frequency $\omega_0 = \sqrt{k/m}$ (isotropic harmonic oscillator). Determine how the angular frequency of oscillations changes if this spatial oscillator is placed in a constant homogeneous magnetic field $\boldsymbol{B} = (0,0, B)$. *Instructions:* Write the equations of motion of the electron. Look for solutions in the x_1x_2 plane in the form $x_i = A_i \exp(i\omega t)$, $i = 1, 2$. To determine ω, use the condition $|eB/2m| \ll \omega_0$. Consider to what extent this condition is met for an electron emitting visible light and placed in a magnetic field ~ 1T.
[In the plane x_1x_2: $\omega \doteq \omega_0 \pm \frac{eB}{2m}$, ve směru osy x_3: $\omega = \omega_0$]

4 *Constant homogeneous magnetic field.* Show that the vector potential $\boldsymbol{A} = \boldsymbol{B} \times \boldsymbol{r}\,/\,2$ determines a constant homogeneous magnetic field $\boldsymbol{B}$. When selecting a coordinate system such that $\boldsymbol{B} = (0,0, \boldsymbol{B})$, where $\boldsymbol{B}$ = const, specify: a) Cartesian components A_x, A_y, A_z, b) components A_R, A_φ, A_z in cylindrical coordinates R, φ, z. *Instructions:* Components A_R, A_φ, A_z are given by orthogonal projections of $\boldsymbol{A}$ into the unit vectors $\boldsymbol{e}_R$, $\boldsymbol{e}_\varphi$, $\boldsymbol{e}_z$ in the directions of the coordinate curves, i.e. in the directions $\partial\boldsymbol{r}/\partial R$, $\partial\boldsymbol{r}/\partial\phi$, $\partial\boldsymbol{r}/\partial z$.
[a) $\boldsymbol{A} = (-By/2, Bx/2, 0)$, b) $A_R = A_z = 0,\ A_\varphi = BR/2$]

5 *Constant homogeneous magnetic field.* Characterize the class of vector potentials $\boldsymbol{A}$, which determine the constant homogeneous magnetic field $\boldsymbol{B} = (0,0, B)$, B = const. Do they include $\boldsymbol{A}' = (-By, 0,0)$?
[$\boldsymbol{A}' = (-By/2, Bx/2, 0) + \mathrm{grad}\Lambda(x, y, z),\ \Lambda = Bxy/2$]

6 *Preserving quantities in a constant magnetic field.* The vector potential $\boldsymbol{A} = \boldsymbol{B} \times \boldsymbol{r}\,/\,2$ determines a constant homogeneous magnetic field $\boldsymbol{B}$. Write a Hamiltonian $H\,(\boldsymbol{r}, \boldsymbol{p})$ of a particle a mass of mass m with charge q in this magnetic field. Prove that vector quantity $\boldsymbol{p} + q\boldsymbol{A}$ represents three integrals of motion of this system. *Instructions:* Select the axis z in the direction of the magnetic field.

7 Show that when a relativistic particle with charge q and rest mass m_0 is moving in an external magnetic field determined by vector potential $\boldsymbol{A} = (0,0, A(x, y))$, the quantity $p_z = \frac{m_0 v_z}{\sqrt{1-\frac{v^2}{c^2}}} + qA(x,y).$ is preserved.

Instructions: Determine the canonical momentum corresponding to the cyclic coordinate z.

8 *Magnetic flux through the surface.* Use the Stokes theorem to specify the physical meaning of the line $\oint_\Gamma \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}$, d$\boldsymbol{l}$, where $\boldsymbol{A}$ is the vector potential of a magnetic field and Γ is a closed curve bounding a two-dimensional surface f. What physical quantity does this integral determine? Is this physical quantity gauge-invariant?
[$\oint\limits_\Gamma \boldsymbol{A}.\mathrm{d}\boldsymbol{l} = \int\limits_f \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{f}$]

9 *Singular magnetic field.* Let the vector potential in the area of space R^3, where $x \neq 0$, $y \neq 0$, have components

$$A_x = -\frac{\Phi}{2\pi}\frac{y}{x^2 + y^2}, \qquad A_y = \frac{\Phi}{2\pi}\frac{x}{x^2 + y^2}, \qquad A_z = 0.$$

a) Determine the magnetic field in this area. b) Calculate the line integral $\oint_\Gamma \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}$, of this vector potential along the circle $x^2 + y^2 = r^2 \neq 0$. Explain which singular magnetic field this vector potential corresponds to. *Instructions:* Integrate in polar coordinates.

[a) $\boldsymbol{B} = \boldsymbol{0}$, b) Φ]

10 *Coulomb potential in anisotropic medium.* Determine the electrostatic potential field induced by a point charge e located at the origin in a homogeneous anisotropic dielectric medium with relative permittivity tensor ε_{ik}, $D_i = \varepsilon_0 \sum_k \varepsilon_{ik} E_k$. *Instructions:* Derive the Poisson equation in the principal axes of the tensor ε_{ik}. By substituting $x_i = x_i' \sqrt{\varepsilon_i}$ transform into the usual form of the Poisson equation.

$$\left[\Delta' \varphi(\boldsymbol{r}') = -\frac{e}{\varepsilon_0 \sqrt{\varepsilon_1 \varepsilon_2 \varepsilon_3}} \delta^3(\boldsymbol{r}') \qquad \Rightarrow \qquad \varphi(\boldsymbol{r}) = \frac{e}{\varepsilon_0 \sqrt{\varepsilon_1 \varepsilon_2 \varepsilon_3}} \frac{1}{\sqrt{\frac{x_1^2}{\varepsilon_1} + \frac{x_2^2}{\varepsilon_2} + \frac{x_3^2}{\varepsilon_3}}} \right.$$

and in the general Cartesian coordinates with the matrix $\varepsilon = (\varepsilon_{ik})$ we can write

$$\left. \varphi(\boldsymbol{r}) = \frac{e}{4\pi\varepsilon_0 \sqrt{|\varepsilon| \sum\limits_{ik} (\varepsilon^{-1})_{ik} x_i x_k}}. \right]$$

11 *The electric dipole moment.* Electric dipole moment of the charges distributed in the bounded volume V with the density $\rho(\boldsymbol{r}, t)$ is $\rho(t) = \int_V \boldsymbol{r}\rho\,(\boldsymbol{r}, t)\mathrm{d}V$. a) Determine the electric dipole moment of the point charge system at points $r_\alpha(t)$, $\alpha = 1, \dots,$ n. b) Prove that the electric dipole moment of a neutral charge system does not depend on the origin selection. *Instructions:* Translate the coordinate system by the formula $\boldsymbol{r}' = \boldsymbol{r} + \boldsymbol{a}$. c) Prove that the electric dipole moment of the point symmetric charge distribution $\rho\,(-\boldsymbol{r}) = \rho(\boldsymbol{r})$ is equal to zero.

[a) $\boldsymbol{p}(t) = \sum\limits_{\alpha} e_\alpha \boldsymbol{r}_\alpha(t)$]

12 *Stationary current.* Show that in the case of stationary currents div$\boldsymbol{j} = 0$ in bounded volume V, $\int \boldsymbol{j}\mathrm{d}V = 0$ is valid. *Instructions:* Divide the current into closed current loops for which $\int \boldsymbol{j}\mathrm{d}V = \oint \mathrm{Idl} = 0$

13 *The magnetic dipole moment* of current distribution $\boldsymbol{j}(\boldsymbol{r}, t)$ in the finite volume V is defined by the relation:

$$\boldsymbol{m}(t) = \tfrac{1}{2} \int_V \boldsymbol{r} \times \boldsymbol{j}(\boldsymbol{r}, t)\mathrm{d}V.$$

a) What is the magnetic dipole moment of the system N of point charges e_α located at points $\boldsymbol{r}_\alpha(t)$ and moving at velocities $\boldsymbol{v}_\alpha(t)$? b) What is the magnetic dipole moment of the closed plane curve Γ with linear current I flowing though it? *Instructions: $\boldsymbol{j}\, dV = I\, dl$.*

[a) $\boldsymbol{m}(t) = \frac{1}{2} \sum\limits_{\alpha=1}^{N} e_\alpha \boldsymbol{r}_\alpha(t) \times \boldsymbol{v}_\alpha(t)$, b) $\boldsymbol{m} = \frac{I}{2} \oint_\Gamma \boldsymbol{r} \times \mathrm{d}\boldsymbol{l} = IS\boldsymbol{n}$,
where S is the surface bounded by a loop and n is its normal.]

14 *Pulling a dielectric medium between capacitor plates* Calculate how the energy of the electrostatic field will change, if we fill the capacitor space with a homogeneous soft dielectric medium. *Instructions:* compare the solution of Maxwell's electrostatic equations in vacuum and in a dielectric medium.

15 *Force in the capacitor* Plate capacitor consists of two parallel conductive plates with area S carrying charges $+Q$ and $-Q$. Plates are placed in a dielectric medium with permittivity ε. If the plates are large and their distance is small, then the electric field is concentrated practically between the plates and is homogeneous. Calculate the forces with which the plates interact. *Instructions:* Calculate the capacitor field energy and its change when the plates spacing changes: a) if the plates are isolated and their charge is constant, b) if the plates have a constant potential difference φ_0. Based on this, calculate the forces.

[a) $Q =$konst.: $(\mathrm{d}W)_a = Q^2 \mathrm{d}l/(2\varepsilon S)$, b) $\varphi_0 =$konst.: $(\mathrm{d}W)_b = -\frac{1}{2}\varepsilon S \varphi_0^2 \mathrm{d}l/l^2$;
$(\mathrm{d}W)_a = -(\mathrm{d}W)_b$, $F_a/S = -F_b/S = \varepsilon E^2/2$]

16 *Maxwell stress tensor* Analyze the meaning of Maxwell stress tensor of electrostatic field in a homogeneous soft dielectric medium. Using the relation (1.54), show that in the direction of field E, the dielectric medium is subjected to tensile stress $\varepsilon E^2/2$, while in the direction perpendicular to $\boldsymbol{E}$, it is subjected to pressure stress of the same magnitude. (Analogous results also apply to a constant magnetic field.)

$$\left[\boldsymbol{F} = \oint\limits_{\partial V} [\varepsilon \boldsymbol{E}(\boldsymbol{E} \cdot \boldsymbol{n}) - \frac{1}{2}\varepsilon \boldsymbol{E}^2 \boldsymbol{n}]\mathrm{d}f; \right.$$

in cases E || d***f*** and ***E*** ⊥ d***f***, where d***f*** = ***n***d*f*, we obtain

$$\boldsymbol{F}_{\parallel} = \oint_{\partial V} \frac{1}{2}\varepsilon E^2 \boldsymbol{n}\mathrm{d}f = -\boldsymbol{F}_{\perp}.\Bigg]$$

7 *Electromagnetic radiation of the Sun* near the Earth has an intensity I ≐ 1365 W m^{-2} called solar irradiance (formerly a solar constant). Based on this, calculate the mean values of the magnitude of the electric field and the magnetic field in the electromagnetic field of solar radiation near the Earth. These are the quantities $E = \sqrt{\langle E^2 \rangle}$ I $B = \sqrt{\langle B^2 \rangle}$ $(\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{N\ A^{-2}})$.
[$510\ \mathrm{V/m},\ 1,7 \cdot 10^{-6}\ \mathrm{T}$]

18 *Potentials of a moving charge.* Calculate the potentials of the electromagnetic field induced in vacuum by a point particle with charge *q*, which moves uniformly in a straight line at the velocity ***V*** = (*V*, 0,0) in the inertial system S of an observer. What is the form of the equipotential surfaces of the scalar potential? *Instructions:* Use the rest system *S′* of a particle located at the origin of the system *S′* and the special Lorentz transformation *S′* → *S* of the four-potential $A^\mu = (\varphi/c, \boldsymbol{A})$.

$$\left[\ \varphi = \frac{q}{4\pi\varepsilon_0\sqrt{(x-Vt)^2 + (1-\beta^2)(y^2+z^2)}}, \quad \boldsymbol{A} = (\boldsymbol{V}/c^2)\varphi\ \right]$$

19 Field of a *moving charge* Calculate the vectors ***E***(*x, y, z, t*), ***B***(*x, y, z, t*) of the electromagnetic field in the inertial system *S* of the observer, if it is induced in vacuum by a point particle with charge *q*, which is moving uniformly in a straight line at the velocity ***V*** = (*V*, 0, 0) in the system *S. Instructions:* Use the formula for the Coulomb electric field **E'** induced in the inertial system *S′* by a particle located in the beginning of the system *S′*, and a special Lorentz transformation *S′* → *S* of the electromagnetic field tensor components $F'^{\mu\nu}(x'^{\lambda})$.

$$\left[\ \boldsymbol{E} = \frac{q(1-\beta^2)(\boldsymbol{r} - \boldsymbol{V}t)}{4\pi\varepsilon_0[(x-Vt)^2 + (1-\beta^2)(y^2+z^2)]^{3/2}}, \quad \boldsymbol{B} = \frac{\boldsymbol{V}}{c^2} \times \boldsymbol{E}\ \right]$$

ELECTROMAGNETIC WAVES - Problems

1 *Lines of force of electric intensity.* Start from the differential equation of the force lines $\boldsymbol{E} \times \mathrm{d}\boldsymbol{r} = 0$ and derive an implicit equation for a single-parameter system of force lines of the field generated by a system of point charges lying on the x-axis. In the special case when the system consists only of two charges $e_1 = 2$, $x_1 = -a$ and $e_2 = -1$, $x_2 = a$, formulate the equation of the line of force that originates from the charge e_1 at an angle α with an x-axis. At what angle β does this line of force enter the charge e_2? Specify the minimum and maximum value, and such that the lines of force still end up in the charge e_2. What is the angle α of the infinite line of force originating from the charge e_1 and directed perpendicularly to the x-axis? Formulate the equation of its asymptote. *Instruction:* Integrating factor of the differential equation of the lines of force $E_y\mathrm{d}x - E_x\mathrm{d}y = 0$ for charges e_i in points x_i of the x-axis is $\mu(x, y) = y$.

[$\sum_i e_i \cos\vartheta_i = C$, kde $\cos\vartheta_i = \frac{x-x_i}{\sqrt{(x-x_i)^2+y^2}}$]

2 *Refraction of force lines of electric intensity.* Determine the law of refraction of force lines of electric intensity $\boldsymbol{E}$ and the surface density η_P of a charge bound on the boundary of two soft dielectric mediums ε, μ. *Instructions:* $E_{1t} - E_{2t} = 0$, $\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = 0$

[$\frac{\mathrm{tg}\vartheta_1}{\mathrm{tg}\vartheta_2} = \frac{E_{1t}/E_{1n}}{E_{2t}/E_{2n}} = \frac{\varepsilon_1}{\varepsilon_2}$; $\eta_P = E_{1n} - E_{2n} = \frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2}E_{1n}$]

3 *Linear polarization.* What is a complex notation of a monochromatic plane wave propagating in the soft medium ε, μ in the direction of the positive axis z? The wave is linearly polarized in the direction of the x-axis.

[$\widehat{\boldsymbol{E}} = (E_0 e^{i(kz-\omega t)}, 0, 0)$, $\widehat{\boldsymbol{B}} = (0, \frac{E_0}{v}e^{i(kz-\omega t)}, 0)$]

4 *Circular polarization.* Write the complex expression for a monochromatic plane wave propagating in a dispersed medium with the refractive index n in the direction of the negative x-axis. The wave is circularly polarized in the right-hand direction.

[$\widehat{\boldsymbol{E}} = (0, E_0 \exp[-i\omega(\frac{n}{c}x + t)], E_0 \exp[-i\omega(\frac{n}{c}x + t) - \frac{\pi}{2}])$,
$\widehat{\boldsymbol{B}} = (0, -n\frac{E_0}{c}\exp[-i\omega(\frac{n}{c}x + t) - \frac{\pi}{2}], n\frac{E_0}{c}\exp[-i\omega(\frac{n}{c}x + t)])$]

5 *Vector potential of an electromagnetic wave* A monochromatic plane wave propagates in a dispersed medium in the direction of a positive axis z. Calculate its vector potential in Coulomb gauge if the wave is a) linearly polarized, b) circularly polarized.

6 [a) $\widehat{\boldsymbol{A}} = \frac{E_0}{\omega}e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t-\frac{\pi}{2})}$; b) $\boldsymbol{A} = (\frac{E_0}{\omega}\sin(kz-\omega t), \pm\frac{E_0}{\omega}\cos(kz-\omega t), 0)$] cos($\boldsymbol{k} \cdot \boldsymbol{r} - \omega t$). Show that magnetic field $\boldsymbol{B}$ is automatically transverse, while transverse $\boldsymbol{E}$ requires that $b = \omega\boldsymbol{k} \cdot \boldsymbol{a}/k^2$. Show that under this condition the vector $\boldsymbol{B}$ will be perpendicular to $\boldsymbol{E}$.

7 Determine the calibration transformation that transforms the potentials from the example 8.6 ($b = \omega\boldsymbol{k} \cdot \boldsymbol{a}/k^2$) to the form $\boldsymbol{A}' = \boldsymbol{a}'\cos(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t)$, $\varphi = 0$ corresponding to the Coulomb gauge where $\boldsymbol{k} \cdot \boldsymbol{a}' = 0$.

[$\Lambda = -\frac{\boldsymbol{k}\cdot\boldsymbol{a}}{k^2}\cos(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)$, $\boldsymbol{a}' = \boldsymbol{a} - (\boldsymbol{k}\cdot\boldsymbol{a})\boldsymbol{k}/k^2$]

8 ansformation of the field from the example 8.7 with the function $\Lambda' = \mathbf{r} \cdot \mathbf{A}'$ and show that new potentials are $\boldsymbol{A}'' = (\boldsymbol{r} \cdot \boldsymbol{a}')\boldsymbol{k}\sin(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t)$, $\varphi'' = -\boldsymbol{r} \cdot \boldsymbol{E}(t)$. Note that the field $\boldsymbol{A}''$ is longitudinal and φ'' is gauge-invariant; although φ'' looks like an electrostatic field, it generally depends on time. The order of magnitude $\boldsymbol{A}''$ differs from $\boldsymbol{A}'$ by the factor kr; show that in the case of light incident on atom, $kr \approx 10^{-5}$. If the electromagnetic wave falls is incident on an atom, the whole interaction in Hamilton's function can be expressed with good accuracy only by the potential $U = e\varphi = -e\boldsymbol{r} \cdot \boldsymbol{E}$ (for each electron) and the influence of the magnetic field can be neglected.

9 Show that vector potential $\boldsymbol{A}''$ from the example 8.8 leads to the same magnetic field as the original $\boldsymbol{A}$ form the example 8.6. Explain how is it possible that the vector potential $\boldsymbol{A}''$, which in many described cases is much smaller than $\boldsymbol{A}$, describes the same field?

10 Show that the time mean value of Poynting vector ($T = 2\pi/\omega$) can be calculated in the complex notation by the formula $\langle\boldsymbol{S}\rangle = \frac{1}{2}\mathrm{Re}\,(\hat{\boldsymbol{E}} \times \hat{\boldsymbol{H}}^*)$, where the asterisk denotes complex conjugate vector. Use Re $\hat{a} = (\hat{a} + \hat{a}^*)/2$.

11 *Electromagnetic wave on the boundary* A monochromatic plane wave is incident from vacuum to the surface of a homogeneous dielectric medium ($\varepsilon \neq \varepsilon_0$, $\mu = \mu_0$) at an angle ϑ_1. Its direction of polarization forms an angle Θ with the plane of incidence.

Calculate reflectivity $R = I_1/I_0$ and permeability $T = I_2/I_0$.

$$[\mathcal{R} = \mathcal{R}^{\|}\cos^2\Theta + \mathcal{R}^{\perp}\sin^2\Theta,$$
$$\mathcal{T} = \mathcal{T}^{\|}\cos^2\Theta + \mathcal{T}^{\perp}\sin^2\Theta, \quad \mathcal{T}^{\|} = \sqrt{\varepsilon}(T_E^{\|})^2, \quad \mathcal{T}^{\perp} = \sqrt{\varepsilon}(T_E^{\perp})^2\,]$$

13 Determine the polarizationof the reflected andrefracted waves from the problem 8.11.

Instructions: $E_1^{\|} = R_E^{\|}E_0\cos\Theta,\ E_1^{\perp} = R_E^{\perp}E_0\sin\Theta,$
$E_2^{\|} = T_E^{\|}E_0\cos\Theta,\ E_2^{\perp} = T_E^{\perp}E_0\sin\Theta.$

[Reflected and refracted waves are linearly polarized in directions in directions forming angles to the plane of incidence

$\mathrm{tg}\Theta_1 = (R_E^{\perp}/R_E^{\|})\mathrm{tg}\Theta,\ \mathrm{tg}\Theta_2 = (T_E^{\perp}/T_E^{\|})\mathrm{tg}\Theta$]

14 Determine the solution to the problem 8.11, when the incident wave is unpolarized. To do this, average out the result of the problem 8.11 over all angles Θ.

[$\mathcal{R} = \frac{1}{2}(\mathcal{R}^{\|} + \mathcal{R}^{\perp}),\ \mathcal{T} = \frac{1}{2}(\mathcal{T}^{\|} + \mathcal{T}^{\perp})$]

2.14 Based on the result of the problem 8.11, show that the law of conservation of energy is fulfilled when the electromagnetic wave reflects and passes through the boundary. To do this, prove that the eq $\mathcal{I}_0\cos\vartheta_1 = \mathcal{I}_1\cos\vartheta_1 + \mathcal{I}_2\cos\vartheta_2.$ is valid.

15 Calculate the pressure acting on the surface of a dielectric medium by the electromagnetic wave from the problem 8.11.

[$p = w_0(\cos\vartheta_1 + \mathcal{R}\cos\vartheta_1 - \frac{\mathcal{T}}{n}\cos\vartheta_2),\ w_0 = \mathcal{I}_0/c$]