Czech labour market through the lens of a search and matching DSGE model

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Abstract. This contribution reveals some important structural properties of the Czech labour market in the last fifteen years and evaluates possible changes within this period. A small search and matching model incorporated into standard macroeconomic dynamic stochastic general equilibrium model is estimated using Bayesian techniques. The results show that search and matching aspect provides satisfactory description of employment flows in the Czech economy. Model estimates provide convincing evidence that wage bargaining process is determined mainly by the power of the unions and that the institutional changes of the Czech labour market in the last fifteen years had only little real impact on the matching effectiveness.

Keywords: search and matching model, Bayesian estimation, DSGE model, structural changes

JEL classification: C51, E24, J60
AMS classification: 91B40, 91B51

1 Introduction

The goal of my contribution is to reveal some interesting and important structural properties of the Czech labour market in the last fifteen years and to evaluate possible changes within this period. For this purpose, I use a small search and matching model incorporated into standard macroeconomic dynamic stochastic general equilibrium model (DSGE). Search and matching model is an important tool to model labour market dynamics. This model is a log-linear version of the model originally developed by Lubik [4]. Using real macroeconomic data I am able to estimate some key labour market indicators: the wage bargaining power of unions, the match elasticity of unemployed and the efficiency of the matching process.

The structure of my contribution is as follows. The next section provides a short description of the small search and matching DSGE model which is used for my analysis. Section 3 discusses used data, priors and estimation techniques. Section 4 presents the main results. Section 5 provides a deeper insight into model properties and its ability to match observed data. Section 6 concludes this contribution with some ideas regarding the possibilities of further research in this area.

2 The model

As mentioned previously, I use the model developed by Lubik [4]. It is a simple search and matching model incorporated within a standard DSGE framework. The labour market is subject to friction because a time-consuming search process for workers and firms. The wages are determined by the outcome of a bargaining process which serves as a mechanism to redistribute the costs of finding a partner.

Households Representative household maximizes its expected utility function

$$E_t \sum_{j=1}^{\infty} \beta^{j-t} \left[\frac{C_j^{1-\sigma} - 1}{1-\sigma} - \chi_j n_j \right], \tag{1}$$

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where C is aggregate consumption, $n \in [0,1]$ is a fraction of employed household members (determined by the matching labour market), $\beta \in (0,1)$ is the discount factor and $\sigma \geq 0$ is the coefficient of relative risk aversion. Variable χ_t represents an exogenous stochastic process which may be taken as a labour shock. The budget constraint is defined as

$$C_t + T_t = w_t n_t + (1 - n_t)b + \Pi_t, \tag{2}$$

where b is unemployment benefit financed by a lump-sum tax, T. Variable Π_t are profits from ownership of the firms and w is wage. There is no explicit labour supply because it is an outcome of the matching process. The first-order condition is thus simply

$$C_t^{-\sigma} = \lambda_t, \tag{3}$$

where λ_t is the Lagrange multiplier on the budget constraint.

Labour market The labour market is characterized by search frictions captured by a standard Cobb-Douglas matching function

$$m(u_t, v_t) = \mu_t u_t^{\xi} \nu_t^{1-\xi}, \tag{4}$$

where unemployed job seekers, u_t, and vacancies, ν_t, are matched at rate $m(u_t, \nu_t)$. Parameter $0 < \xi < 1$ is a match elasticity of the unemployed and μ_t is stochastic process measuring the efficiency of the matching process. Aggregate probability of filling a vacancy may be defined as

$$q(\theta_t) = m(u_t, \nu_t)/\nu_t, \tag{5}$$

where $\theta_t = \frac{\nu_t}{u_t}$ is a standard indicator of the labour market tightness. The model assumes that it takes one period for new matches to be productive. Moreover, old and new matches are destroyed at a constant separation rate, $0 < \rho < 1$, which corresponds to the inflows into unemployment. Evolution of employment, $n_t = 1 - u_t$, is given by

$$n_t = (1 - \rho)\left[n_{t-1} + \nu_{t-1} q(\theta_{t-1})\right]. \tag{6}$$

Firms As a deviation from the standard search and matching framework, the model assumes monopolistic firms. Demand function of a firm is defined by

$$y_t = \left(\frac{p_t}{P_t}\right)^{-1-\epsilon} Y_t, \tag{7}$$

where y_t is firm's production (and its demand), Y_t is aggregate output, p_t is price set by the firm, P_t is aggregate price index and ϵ is demand elasticity which will be not treated as a stochastic process in my empirical application. Production function of each firm is

$$y_t = A_t n_t^{\alpha}, \tag{8}$$

where A_t is an aggregate technology (stochastic) process and $0 < \alpha \leq 1$ introduces curvature in production. Capital is fixed and firm-specific. The firm controls the number of workers, n_t, number of posted vacancies, ν_t, and its optimal price, p_t, by maximizing the inter-temporal profit function

$$E_t \sum_{j=1}^{\infty} \beta^{j-t} \lambda_j \left[p_j \left(\frac{p_j}{P_j}\right)^{-(1+\epsilon)} Y_j - w_j n_j - \frac{\kappa}{\psi} \nu_j^{\psi} \right], \tag{9}$$

subject to the employment accumulation equation (7) and production function (8). Profits are evaluated in terms of marginal utility λ_j. The costs of vacancy posting is $\frac{\kappa}{\psi} v_t^{\psi}$, where $\kappa > 0$ and $\psi > 0$. For $0 < \psi < 1$, posting costs exhibit decreasing returns. For $\psi > 1$, the costs are increasing while vacancy costs are fixed for $\psi = 1$. The first-order conditions are

$$\tau_t = \alpha \frac{y_t}{n_t} \frac{\epsilon}{1+\epsilon} - w_t + (1-\rho) E_t \beta_{t+1} \tau_{t+1}, \tag{10}$$

$$\kappa \nu_t^{\psi - 1} = (1-\rho) q(\theta_t) E_t \beta_{t+1} \tau_{t+1}, \tag{11}$$

where $\beta_{t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}$ is a stochastic discount factor and τ_t is the Lagrange multiplier associated with employment constraint. The first condition represents current-period marginal value of a job. The second condition is a link between the cost of vacancy and the expected benefit of a vacancy in terms of the marginal value of a worker (adjusted by the job creation rate, $q(\theta_t)$).

Wage bargaining Wages are determined as the outcome of a bilateral bargaining process between workers and firms. Both sides of the bargaining maximize the joint surplus from employment relationship:

$$S_t \equiv \left(\frac{1}{\lambda_t}\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t}\right)^{\eta}\left(\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t}\right)^{1-\eta}, \tag{12}$$

where $\eta \in [0,1]$ is the bargaining power of workers, $\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t}$ is the marginal value of a worker to the household's welfare and $\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t}$ is the marginal value of a worker to the firm. The term $\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t} = \tau_t$ is given by the first-order condition (10). Recursive representation for $\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t}$ is derived as

$$\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t} = \lambda_t w_t - \lambda_t b - \chi_t + \beta E_t \frac{\partial \mathcal{W}_{t+1}(n_{t+1})}{\partial n_{t+1}}\frac{\partial n_{t+1}}{\partial n_t}. \tag{13}$$

Using employment equation (6), it holds $\frac{\partial n_{t+1}}{\partial n_t} = (1-\rho)[1-\theta_t q(\theta_t)]$. All real payments are valued at the marginal utility λ_t. Standard optimality condition for wages may be derived as

$$(1-\eta)\frac{1}{\lambda_t}\frac{\partial \mathcal{W}_t(n_t)}{\partial n_t} = \eta\frac{\partial \mathcal{J}_t(n_t)}{\partial n_t}. \tag{14}$$

Expression for the bargained wage is given after some algebra as

$$w_t = \eta\left[\alpha\frac{y_t}{n_t}\frac{\epsilon}{1+\epsilon} + \kappa\nu_t^{\psi-1}\theta_t\right] + (1-\eta)\left[b + \chi_t C_t^{\sigma}\right]. \tag{15}$$

Closing the model The model assumes, that unemployment benefits, b, are financed by lump-sum taxes, T, where a condition of balanced budget holds, i.e. $T_t = (1-n_t)b$. Social resource constraint is thus $C_t + \frac{\kappa}{\psi}\nu_t^{\psi} = Y_t$. The technology shock A_t, the labour shock χ_t and the matching shock μ_t are assumed to be independent $AR(1)$ processes (in logs) with coefficients ρ_i, $i \in (A, \xi, \mu)$ and innovations $\epsilon_t^i \sim N(0, \sigma_i^2)$.

Log-linearised model For estimation purposes, I did not use the non-linear form of the model mentioned in the previous section (of course, this form is important to understand the meaning of the key structural model parameters). Instead of that, I use a log-linear version of the model based on my own derivations.[1] In the following equations, the line over a variable means its steady-state value (derived simply from the non-linear equations).[2] The variables with a tilde represent the gaps from their steady-states.

$$\tilde{\lambda}_t = -\sigma\tilde{C}_t \qquad \tilde{m}_t = \tilde{\mu}_t + \xi\tilde{u}_t + (1-\xi)\tilde{\nu}_t$$

$$\tilde{q}_t = \tilde{m}_t - \tilde{\nu}_t \qquad \tilde{\theta}_t = \tilde{\nu}_t - \tilde{u}_t$$

$$\tilde{n}_t = -\frac{\overline{u}}{1-\overline{u}}\tilde{u}_t \qquad \tilde{n}_t = \frac{1}{\overline{n}+\overline{vq}}\left[\overline{u}\tilde{n}_{t-1} + \overline{q}\overline{\nu}(\tilde{\nu}_{t-1} + \tilde{q}_{t-1})\right]$$

$$\tilde{y}_t = (-1-\epsilon)(\tilde{p}_t - \tilde{P}_t) + \tilde{Y}_t \qquad \tilde{y}_t = \tilde{A}_t + \alpha\tilde{n}_t$$

$$\tilde{\tau}_t = \frac{1}{\alpha\frac{\overline{y}}{\overline{n}}\frac{\epsilon}{1+\epsilon}\overline{w} + (1-\rho)\overline{\beta}\overline{\tau}}\left[\alpha\frac{\epsilon}{1+\epsilon}(\tilde{y}_t - \tilde{n}_t) - \overline{w}\tilde{w}_t + (1-\rho)\overline{\tau}\overline{\beta}E_t\left(\tilde{\beta}_{t+1} + \tilde{\tau}_{t+1}\right)\right]$$

$$(\psi-1)\tilde{\nu}_t = \tilde{q}_t + E_t\left(\tilde{\beta}_{t+1} + \tilde{\tau}_{t+1}\right)$$

$$\tilde{\beta}_t = \tilde{\lambda}_t + \tilde{\lambda}_{t-1}$$

$$\tilde{w}_t = \frac{1}{\overline{w}}\left[\eta\left(\alpha\frac{\epsilon}{1+\epsilon}\frac{\overline{y}}{\overline{n}}(\tilde{y}_t - \tilde{n}_t) + \kappa\overline{\nu}^{\psi-1}\overline{\theta}\left((\psi-1)\tilde{v}_t + \tilde{\theta}_t\right)\right) + (1-\eta)\overline{\chi}\overline{C}^{\sigma}(\tilde{\chi}_t + \sigma\tilde{C}_t)\right]$$

$$\tilde{Y}_t = \frac{1}{\overline{C} + \frac{\chi}{\psi}\overline{\nu}^{\psi}}\left(\overline{C}\tilde{C}_t + \kappa\overline{\nu}^{\psi}\tilde{\nu}_t\right)$$

[1] Log-linear version is not a part of the original contribution of Lubik [4].

[2] Initial steady-state values are calibrated as follows: $\mu^* = A^* = \chi^* = 1$, $\beta^* = 0.99$, $u^* = 0.0763$, $\nu^* = 0.0127$. Remaining steady-states are computed using these values and the prior means of all parameters.

$$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_t^A \quad \tilde{\chi}_t = \rho_\chi \tilde{\chi}_{t-1} + \epsilon_t^\chi \quad \tilde{\mu}_t = \rho_\mu \tilde{\mu}_{t-1} + \epsilon_t^\mu \quad \tilde{Y}_t = \rho_Y \tilde{Y}_{t-1} + \epsilon_t^Y$$

The last equation results from the fact that variable $\tilde{Y}$ is an observed variable. We have thus four shocks (ϵ_t^i for four observed variables – $\tilde{u}$, $\tilde{\nu}$, $\tilde{w}$ and $\tilde{Y}$). The model consists of 17 endogenous variables (variable $(\tilde{p}_t - \tilde{P}_t)$ is a single variable in my application), four shocks and 14 parameters.

3 Data and priors

The model for the Czech economy is estimated using the quarterly data set covering a sample from 1996Q1 to 2010Q4. The observed variables are real output (GDP, in logs), hourly earnings (in logs), unemployment rate and rate of unfilled job vacancies. All data are seasonally adjusted and de-trended (excluding vacancies) using Hodrick-Prescott filter (with the smoothing parameter $\lambda = 1600$). The rate of unfilled job vacancies was demeaned prior estimation. The variables used are expressed as corresponding gaps. The original data are from databases of the OECD and the Czech Statistical Office (CZSO).[3] In the following parts, I will not use the mark ˜ to explicitly express the appropriate gaps.

Description	Parameter	Density	Mean	Std. Dev.
Discount factor	β	Fixed	0.99	—
Labor elasticity	α	Fixed	0.67	—
Demand elasticity	ϵ	Fixed	10	—
Relative risk aversion	σ	Gamma	1.00	0.50
Match elasticity	ξ	Gamma	0.70	0.10
Separation rate	ρ	Gamma	0.10	0.05
Bargaining power of the workers	η	Uniform	0.50	0.3
Unemployment benefits	b	Beta	0.20	0.15
Elasticity of vacancy creation cost	ψ	Gamma	1.00	0.50
Scaling factor on vacancy creation cost	κ	Gamma	0.10	0.05
AR coefficients of shocks	$\rho_{\{\chi,A,\mu,Y\}}$	Beta	0.8	0.2
Standard deviation of shocks	$\sigma_{\{\chi,A,\mu,Y\}}$	Inv. Gamma	0.05	∞
Standard deviation of measurement errors	$\sigma^*_{\{u\}}$	Uniform	0.001	0.0006
Standard deviation of measurement errors	$\sigma^*_{\{w,\nu\}}$	Uniform	0.001	0.0003

Table 1: Parameters description and prior densities

Parameters are estimated using Bayesian techniques combined with Kalman filtering procedures. All computations have been performed using Dynare toolbox [2] for Matlab. Table 1 reports the model parameters and the corresponding prior densities. The priors (and calibrations) are similar to those used by Lubik [4]. On the other hand, the standard deviations are rather uninformative.

4 Estimation results

Table 2 presents the posterior estimates of parameters and 90% highest posterior density intervals. It may be seen (in comparison with the Table 1) that most of the parameters are moved considerably from their prior means. The data seems to be strongly informative. There are some remarkable results which should be emphasized:

- The first surprising estimate is the bargaining power of workers, η. The mean value of this parameter is almost 0.9 with a 90 percent coverage region that is shifted considerably away from the prior density. This implies that the workers can gain the most of their entire surplus. The firms are thus not willing to create vacancies. This result is in sharp contrast to the results of Lubik [4] or Yashiv [6] who aimed to model the U.S. labour market.

[3] GDP at purchaser prices, constant prices 2000, s.a., CZSO, millions of CZK; index of hourly earnings (manufacturing), 2005=100, s.a., OECD; registered unemployment rate, s.a., OECD; unfilled job vacancies, level (transformed to ratio of unfilled vacancies to labour force), s.a., OECD.

	Posterior mean	90% HPDI			Posterior mean	90% HPDI	
σ	1.0399	0.6036	1.4894	ρ_μ	0.8281	0.7298	0.9525
ξ	0.6459	0.5823	0.7099	ρ_Y	0.8695	0.8197	0.9241
ρ	0.0268	0.0086	0.0486	σ_χ	0.0162	0.0108	0.0214
η	0.8694	0.8311	0.9082	σ_A	0.0080	0.0067	0.0093
b	0.2924	0.1318	0.4688	σ_μ	0.0066	0.0059	0.0073
ψ	3.5706	3.0908	3.9944	σ_Y	0.0095	0.0080	0.0108
κ	0.0880	0.0637	0.1199	σ_u^*	0.0005	0.0002	0.0009
ρ_χ	0.7831	0.7208	0.8555	σ_w^*	0.0003	0.0001	0.0004
ρ_A	0.9629	0.9332	0.9996	σ_ν^*	0.0002	0.0001	0.0004

Table 2: Parameter estimates

- The second interesting result is the estimated separation rate, ρ. This parameter is considerably lower than the one estimated by Lubik [4]. Its value supports the view of less flexible Czech labour market with limited ability to destroy old and new matches.
- The third remarkable estimate is the vacancy posting elasticity, ψ. The posterior mean 3.57 is shifted away from the prior mean. The vacancy creation is thus more costly because of increasing marginal posting costs (increasing in the level of vacancies or labour market tightness, θ). Lubik [4] estimated this parameter at the mean value of 2.53. In this case, the high value of ψ may be interpreted as a balancing factor which "restrict" potentially excessive vacancy creation driven by the low bargaining power. In case of the Czech labour market, this higher value provides further evidence of specifically less flexible labour market.
- The estimate of parameter b corresponds to a reasonable value 0.3 which might be in accordance with the real unemployment benefits paid within the Czech social insurance system (30% of average wage).
- The posterior mean of the matching function parameter, ξ, is in accordance with the common values in literature (see Lubik [4] or Christoffel et al. [1]).

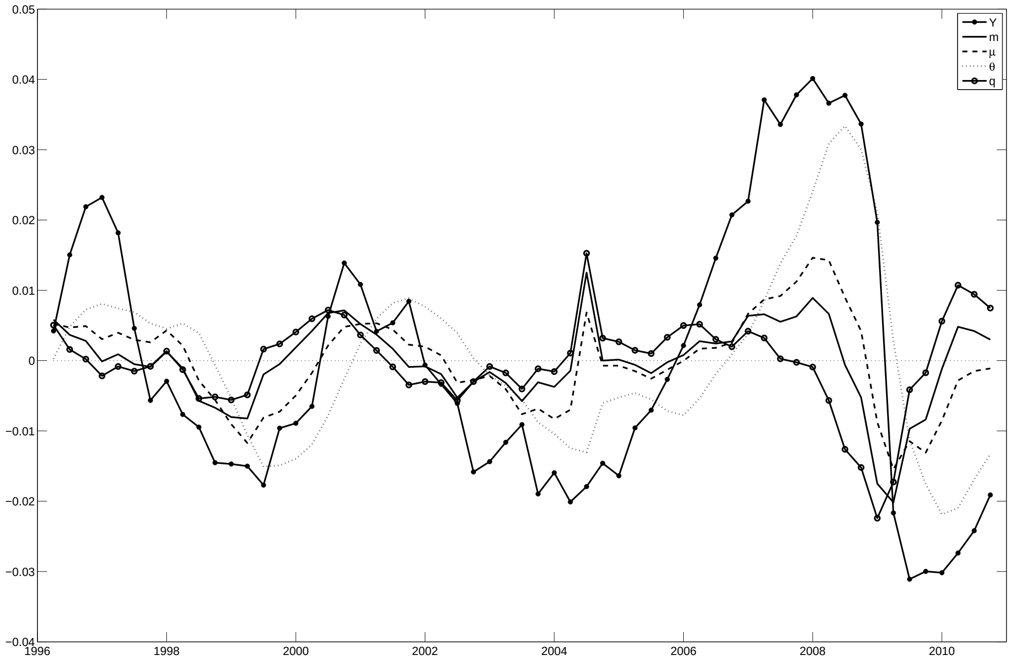

Figure 1: Trajectories of selected (smoothed) variables

Figure 1 presents the trajectories of selected smoothed variables. We can see a relative smooth development of variable q (probability of filling a vacancy) with a sharp decline at the end of the year 2007. This evidence is in favour of conclusion presented by Němec and Vašíček [5] who stressed the role of an obvious lack of employees in the Czech economy. This tendency was reverted as a result of the last global economic slowdown starting at the end of 2008. The efficiency of the matching process is strongly correlated with the output gap (correlation coefficient is 0.8). This indicator is thus seemingly independent of the institutional framework of the Czech labour market. It probably means that the changes in labour market institution have been mostly marginal (with little real impacts). The correlation between output gap and the matching variable m is relative small, 0.4. On the other hand, the correlation between actual value of output gap and the lagged value of m is 0.65. Current value of matching function might be thus an useful indicator of future (one-quarter ahead) changes in the real output (output gap).

5 Model evaluation

In order to see how the model fits the data, sample moments, autocorrelation coefficients and cross-correlations are computed. I computed these statistics from simulation of the estimated models with parameters set at their posterior means. All these statistics correspond to the four observed series (unemployment gap, u, gap of vacancies, ν, gap of the wages, w, and output gap, Y). The results may be found in the Tables 3 and 4.

		Sample moments		Lags for autocorrelation coefficients			
		Mean	Std. dev.	1	2	3	4
u	data	0.00	0.009	0.91	0.71	0.45	0.16
	model	−0.00	0.010	0.88	0.70	0.51	0.35
	90% HPDI	(−0.01, 0.01)	(0.007, 0.014)	(0.79, 0.94)	(0.48, 0.83)	(0.11, 0.72)	(−0.08, 0.62)
ν	data	0.00	0.004	0.91	0.71	0.45	0.17
	model	0.00	0.008	0.72	0.54	0.40	0.29
	90% HPDI	(−0.01, 0.01)	(0.006, 0.011)	(0.55, 0.87)	(0.25, 0.80)	(0.08, 0.73)	(−0.09, 0.67)
w	data	0.00	0.014	0.80	0.53	0.29	0.14
	model	0.00	0.054	0.72	0.52	0.36	0.24
	90% HPDI	(−0.04, 0.04)	(0.041, 0.071)	(0.57, 0.84)	(0.30, 0.72)	(0.06, 0.61)	(−0.09, 0.57)
Y	data	0.00	0.020	0.91	0.74	0.54	0.33
	model	0.00	0.017	0.79	0.62	0.47	0.36
	90% HPDI	(−0.01, 0.01)	(0.012, 0.024)	(0.64, 0.88)	(0.33, 0.77)	(0.09, 0.70)	(0.01, 0.63)

Table 3: Sample moments and autocorrelation coefficients

	Data				Model (90% HPDI)			
	u	ν	w	Y	u	ν	w	Y
u	1.00	−0.90	−0.74	−0.77	1.00	−0.17	−0.12	−0.16
					(1.00, 1.00)	(−0.56, 0.25)	(−0.53, 0.31)	(−0.66, 0.34)
ν	−0.90	1.00	0.80	0.88	−0.17	1.00	0.63	0.72
					(−0.56, 0.25)	(1.00, 1.00)	(0.25, 0.89)	(0.46, 0.87)
w	−0.74	0.80	1.00	0.60	−0.12	0.63	1.00	0.21
					(−0.53, 0.31)	(0.25, 0.83)	(1.00, 1.00)	(−0.26, 0.62)
Y	−0.77	0.88	0.60	1.00	−0.16	0.78	0.21	1.00
					(−0.66, 0.34)	(0.46, 0.87)	(−0.26, 0.62)	(1.00, 1.00)

Table 4: Matrix of correlation

The model is very successful in matching sample moments and autocorrelation coefficients (they are mostly within the appropriate 90% highest posterior density intervals). This ability is not used to be typical for such a small-scale model. But, there is one exception regarding the fit of sample moments. The model predicts higher volatility in wages. This pattern reveals the necessity of enrichment by a new source of wage rigidity (as suggested by Krause and Lubik [3] or Christoffel et al. [1]).

My results are in accordance with the authors arguing that the model with search and matching frictions in the labour market is able to generate negative correlation between vacancies and unemployment (see Krause and Lubik [3]). Unfortunately, the values of cross-correlation coefficients (see the lowest bounds of HPDI in the Table 4) are not sufficient for the correlations of unemployment and the rest of observable variables. The similar experience may be found in the results for U.S. labour market provided by Lubik [4]. Lubik pointed out that this may be due the presence of matching shock, which can act as a residual in employment and wage equations.

6 Conclusion

In my contribution, I investigated structural properties of the Czech labour market using a simple DSGE framework with labour market rigidities. Two sources of rigidities were implemented: wage bargaining mechanism and "'search and matching"' process matching workers and firms. Estimated model provides satisfactory description of employment flows in the Czech economy. Parameter estimates provide convincing evidence that wage bargaining process is determined mainly by the power of the unions and that the institutional changes of the Czech labour market in the last fifteen years had only little real impact on the matching effectiveness. Unfortunately, the model predicts higher volatility in wages. This pattern reveals the necessity of enrichment by a new source of wage rigidity as proposed by Krause and Lubik [3] or Christoffel et al. [1].

Acknowledgements

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